

Atlas of Lie Groups and Representations



www.liegroups.org

Examples of the Atlas Software

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Atlas of Lie Groups and Representations: www.liegroups.org

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Today: Some examples

Examples of the Atlas Software

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- Constant term of the KLV polynomials
- Cohomological Representations

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- Relation with topology of the unitary dual

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- Visualizing the FPP

But First: Unitarity of Arthur representations for real groups

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 Π_Ψ : the Arthur packet of Ψ defined in ABV.

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Corollary: All Arthur representations are unitary.

Before we begin: AI + atlas

[digression]

Constant term of the KLV polynomials

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sweep	blocks	constant terms
all types rank ≤ 5 (sc + ad, both inner classes)	232	$\{0, 1, 2, 4\}$
rank 6 (A_6, B_6, C_6, D_6, E_6)	132	$\{0, 1, 2, 4, 8\}$
rank 7 (A_7, B_7, C_7, D_7)	116	$\{0, 1, 2, 4, 8\}$
D_8 , all forms	84	$\{0, 1, 2, 4, 8, \mathbf{16}\}$
E_7 sc + ad	18	$\{0, 1, 2, 4, 8\}$
intermediate D -isogenies	84	$\{0, 1, 2, 4\}$
E_8 , all blocks but the big one	4	$\{0, 1, 2, 4\}$
exotic isogenies, half-integral γ	~ 590	$\{0, 1, 2, 4, 8\}$
$PSO(10, 10)$ big block, all columns of length ≥ 32	1	$\{1, 2, 4, 8, 16, \mathbf{32}\}$

Cohomological representations

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So: **compute**: $\wedge^k \mathfrak{p}$ as a **K -module** (applied to L)

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$$\mu = \sum_{i=1}^n a_i \pi_i|_K \quad (\pi_i \text{ tempiric}).$$

Therefore we need to compute:

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So: run over θ -stable parabolics, for each compute $H^k(\mathfrak{g}, K, \pi)$
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atlas> show_g_k_cohomology(SL(2,R))
Levi      u_p  l_p  cohomology
u(1)      1    0   [0,1,0]   {discrete series}
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sl(2,R)   0    2   [1,0,1]   {trivial}
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```
atlas> show_g_k_cohomology(Sp(4,R))
```

Levi	u_p	l_p	cohomology	
u(1).u(1)	3	0	[0,0,0,1,0,0,0]	{discrete series}
u(1).u(1)	3	0	[0,0,0,1,0,0,0]	{discrete series}
u(1).u(1)	3	0	[0,0,0,1,0,0,0]	{discrete series}
u(1).u(1)	3	0	[0,0,0,1,0,0,0]	{discrete series}
su(2).u(1)	3	0	[0,0,0,1,0,0,0]	
su(2).u(1)	3	0	[0,0,0,1,0,0,0]	
sl(2,R).u(1)	2	2	[0,0,1,0,1,0,0]	
sl(2,R).u(1)	2	2	[0,0,1,0,1,0,0]	
sl(2,R).u(1)	2	2	[0,0,1,0,1,0,0]	
sp(4,R)	0	6	[1,0,1,0,1,0,1]	{trivial}

```
atlas> show_g_k_cohomology (Spin(4,1))
```

Levi	u_p	l_p	pi_L	cohomology
u(1).u(1)	2	0	triv	[0,0,1,0,0]
u(1).u(1)	2	0	triv	[0,0,1,0,0]
su(2).u(1)	2	0	triv	[0,0,1,0,0]
su(2).u(1)	2	0	triv	[0,0,1,0,0]
sl(2,R).u(1)	1	2	triv	[0,1,0,1,0]
so(4,1)	0	4	triv	[1,0,0,0,1]

```
atlas> show_g_k_cohomology (SO(4,1))
```

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u(1).u(1)	2	0	triv	[0,0,1,0,0]
su(2).u(1)	2	0	triv	[0,0,1,0,0]
sl(2,R).u(1)	1	2	triv	[0,1,0,0,0]
sl(2,R).u(1)	1	2	det	[0,0,0,1,0]
so(4,1)	0	4	triv	[1,0,0,0,0]
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Topology of the unitary dual

Bader/Glasner/Gorfine/Sauer (see arXiv): can atlas analyze this, and also find minimal p so that the matrix coefficients of π are in L^p ; also what about **Non-unitary** representations with cohomology? What about computing Ext groups?

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```
atlas> bader_report(Sp(4,R))
```

i	i_B	param	unitary	isolated	p_0	Ext ¹	coh degrees
0	0	(x=0, [2,1], [0,0])	true	false	2/1	[4,5]	[3]
1	1	(x=1, [2,1], [0,0])	true	false	2/1	[4,6]	[3]
2	2	(x=2, [2,1], [0,0])	true	false	2/1	[5,11]	[3]
3	3	(x=3, [2,1], [0,0])	true	false	2/1	[6,11]	[3]
4	4	(x=4, [2,1], [1,-1]/2)	true	false	3/1	[0,1,7,8,9]	[2,4]
5	5	(x=5, [2,1], [0,1])	true	false	4/1	[0,2,7,9]	[2,4]
6	6	(x=6, [2,1], [0,1])	true	false	4/1	[1,3,8,9]	[2,4]
7	7	(x=7, [2,1], [2,0])	false	false	-1/1	[4,5,10]	chi=-3
8	8	(x=8, [2,1], [2,0])	false	false	-1/1	[4,6,10]	chi=-3
9	9	(x=9, [2,1], [3,3]/2)	false	false	-1/1	[4,5,6,10,11]	chi=-4
10	10	(x=10, [2,1], [2,1])	true	true	-1/1	[7,8,9]	[0,2,4,6]
11	11	(x=10, [3,2], [2,1])	false	false	-1/1	[2,3,9]	chi=2

The Jantzen Filtration

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This is the filtration that controls the change in signature in the deformation algorithm:

$$\mathrm{sig}(I_{1+\epsilon}) = \mathrm{sig}(I_{1-\epsilon}) + (1-s) \sum_{n \text{ odd}} \mathrm{sig}(\mathrm{gr}_n I)$$

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Computing this is a key step in the atlas unitarity algorithm.

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Theorem (Beilinson/Bernstein)

$$\text{mult}(J(\gamma), \text{gr}_n(I(\tau))) = a_{(\ell(\tau) - \ell(\gamma) - n)/2}$$

“the power of q tells you how far $J(\gamma)$ is shifted from its default level $\ell(\tau) - \ell(\gamma)$ ”

```

atlas> print_graded_composition_series(Sp(4,R).trivial)
final parameter(x=10,lambda=[3,2]/1,nu=[2,1]/1)      length=3
x   lambda      nu      l  ld  Q  filtration
10  [ 3, 2 ]/1  [ 2, 1 ]/1    3  0  +1  [1 . . . ]
9   [ 2, 1 ]/1  [ 3, 3 ]/2    2  1  +1  [. 1 . . ]
6   [ 2, 1 ]/1  [ 0, 1 ]/1    1  2  +1  [0 . 1 . ]
5   [ 2, 1 ]/1  [ 0, 1 ]/1    1  2  +1  [0 . 1 . ]
4   [ 2, 1 ]/1  [ 1, -1 ]/2    1  2  +1  [0 . 1 . ]
3   [ 2, 1 ]/1  [ 0, 0 ]/1    0  3  +q  [. 1 . 0 ]
2   [ 2, 1 ]/1  [ 0, 0 ]/1    0  3  +q  [. 1 . 0 ]
1   [ 2, 1 ]/1  [ 0, 0 ]/1    0  3  +1  [. 0 . 1 ]
0   [ 2, 1 ]/1  [ 0, 0 ]/1    0  3  +1  [. 0 . 1 ]

```

```
final parameter(x=44,lambda=[4,3,1]/1,nu=[3,2,1]/1)      length=6
x   lambda          nu          1  ld  Q      filtration
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44  [ 3, 2, 1 ]/1    [ 3, 2, 1 ]/1    6  0   +1    [1 . . . . . ]
43  [ 3, 2, 1 ]/1    [ 5, 5, 2 ]/2    5  1   +1    [. 1 . . . . . ]
42  [ 3, 2, 1 ]/1    [ 6, 3, 3 ]/2    5  1   +1    [. 1 . . . . . ]
...
16  [ 3, 2, 1 ]/1    [ 1, -1, 2 ]/2    2  4   +2+q    [0 . 1 . 2 . . ]
15  [ 3, 2, 1 ]/1    [ 0, 0, 1 ]/1      1  5   +q      [. 0 . 1 . 0 . ]
14  [ 3, 2, 1 ]/1    [ 0, 0, 1 ]/1      1  5   +q      [. 0 . 1 . 0 . ]
13  [ 3, 2, 1 ]/1    [ 0, 0, 1 ]/1      1  5   +1+q    [. 0 . 1 . 1 . ]
12  [ 3, 2, 1 ]/1    [ 0, 0, 1 ]/1      1  5   +1+q    [. 0 . 1 . 1 . ]
11  [ 3, 2, 1 ]/1    [ 0, 1, -1 ]/2    1  5   +1      [. 0 . 0 . 1 . ]
10  [ 3, 2, 1 ]/1    [ 0, 1, -1 ]/2    1  5   +1      [. 0 . 0 . 1 . ]
9   [ 3, 2, 1 ]/1    [ 1, -1, 0 ]/2    1  5   +1      [. 0 . 0 . 1 . ]
8   [ 3, 2, 1 ]/1    [ 1, -1, 0 ]/2    1  5   +1      [. 0 . 0 . 1 . ]
7   [ 3, 2, 1 ]/1    [ 0, 0, 0 ]/1      0  6   +q^2    [0 . 1 . 0 . 0 ]
6   [ 3, 2, 1 ]/1    [ 0, 0, 0 ]/1      0  6   +q      [0 . 0 . 1 . 0 ]
5   [ 3, 2, 1 ]/1    [ 0, 0, 0 ]/1      0  6   +q^2    [0 . 1 . 0 . 0 ]
...
0   [ 3, 2, 1 ]/1    [ 0, 0, 0 ]/1      0  6   +1      [0 . 0 . 0 . 1 ]
```

The Hodge Filtration

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Saito's mixed Hodge modules + Beilinson-Bernstein: every irreducible $(\mathfrak{g}, K)$ -module carries a canonical

$$0 = F_{a-1}(\pi) \subset F_a(\pi) \subset F_{a+1}(\pi) \subset \cdots$$

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Hodge function $\text{hodge}(\pi) : \widehat{K} \rightarrow \mathbb{Z}[v]$:

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$\text{hodge}(\pi)(\mu)|_{v=1} = \text{mult}(\pi)(\mu)$: set $v = 1$ and you recover branching.

$\text{hodge}(\pi)|_{v=s} = \text{sig}^c(\pi)$: the c -form is the **reduction mod 2**.

$SL(2, \mathbb{R})$: **tempered**

```
atlas> set p=trivial(SL(2,R))*0
atlas> p
Value: final parameter(x=2,lambda=[1]/1,nu=[0]/1)
atlas> show_long (hodge_branch_std(p,8))
```

c	codim	x	lambda	hw	dim	height
+1	0	2	[1]/1	[0]	1	0
+v	1	1	[1]/1	[2]	1	1
+v	1	0	[1]/1	[2]	1	1
+v^2	1	1	[3]/1	[4]	1	3
+v^2	1	0	[3]/1	[4]	1	3
+v^3	1	1	[5]/1	[6]	1	5
+v^3	1	0	[5]/1	[6]	1	5

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+v	1	0	[1]/1	[2]	1	1
+v^2	1	1	[3]/1	[4]	1	3
+v^2	1	0	[3]/1	[4]	1	3
+v^3	1	1	[5]/1	[6]	1	5
+v^3	1	0	[5]/1	[6]	1	5

$$\cdots + v^3[-6] + v^2[-4] + v[-2] + [0] + v[2] + v^2[4] + v^3[6] + \cdots$$

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+1	0	2	[1]/1	[0]	1	0
+v	1	1	[1]/1	[2]	1	1
+v	1	0	[1]/1	[2]	1	1
+v^2	1	1	[3]/1	[4]	1	3
+v^2	1	0	[3]/1	[4]	1	3
+v^3	1	1	[5]/1	[6]	1	5
+v^3	1	0	[5]/1	[6]	1	5

$$\cdots + v^3[-6] + v^2[-4] + v[-2] + [0] + v[2] + v^2[4] + v^3[6] + \cdots$$

Tempered: level k is $u(\mathfrak{g})^k \cdot (\text{lowest } K\text{-type})$.

$SL(2, \mathbb{R})$: $\nu = 1$

```
atlas> set q=trivial(SL(2,R))
atlas> q
Value: final parameter(x=2,lambda=[1]/1,nu=[1]/1)
atlas> show_long (hodge_branch_std(q,8))
```

c	codim	x	lambda	hw	dim	height
+1	0	2	[1]/1	[0]	1	0
+1	1	1	[1]/1	[2]	1	1
+1	1	0	[1]/1	[2]	1	1
+v	1	1	[3]/1	[4]	1	3
+v	1	0	[3]/1	[4]	1	3
+v^2	1	1	[5]/1	[6]	1	5
+v^2	1	0	[5]/1	[6]	1	5

```
atlas> set q=trivial(SL(2,R))
atlas> q
Value: final parameter(x=2,lambda=[1]/1,nu=[1]/1)
atlas> show_long (hodge_branch_std(q,8))
```

c	codim	x	lambda	hw	dim	height
+1	0	2	[1]/1	[0]	1	0
+1	1	1	[1]/1	[2]	1	1
+1	1	0	[1]/1	[2]	1	1
+v	1	1	[3]/1	[4]	1	3
+v	1	0	[3]/1	[4]	1	3
+v^2	1	1	[5]/1	[6]	1	5
+v^2	1	0	[5]/1	[6]	1	5

$$\cdots + v^3[-6] + v^2[-4] + v[-2] + v[0] + v[2] + v^2[4] + v^3[6] + \cdots$$

$\mathrm{Sp}(4, \mathbb{R})$: $\nu = 0$

```
atlas> p
Value: final parameter(x=10,lambda=[2,1]/1,nu=[0,0]/1)
atlas> show_long (hodge_branch_std(p,10))
```

c	codim	x	lambda	hwt	dim	height
+1	0	10	[2, 1]/1	[0, 0]	1	0
+v ²	2	4	[1, 0]/1	[1, 1]	3	2
+v ²	3	3	[1, 0]/1	[-2, 2]	1	3
+v ²	3	2	[1, 0]/1	[2, -2]	1	3
+v+v ³	3	1	[1, 0]/1	[0, 2]	3	3
+v+v ³	3	0	[1, 0]/1	[2, 0]	3	3
+v ⁴	2	6	[2, 1]/1	[-1, 3]	3	6
+v ⁴	2	5	[2, 1]/1	[3, -1]	3	6
+v ² +2v ⁴	2	4	[2, 1]/1	[2, 2]	5	6
+v ³ +v ⁵	3	1	[2, 1]/1	[1, 3]	5	7
+v ³ +v ⁵	3	0	[2, 1]/1	[3, 1]	5	7
+v ³ +v ⁵	3	3	[3, 0]/1	[-2, 4]	3	9
+v ³ +v ⁵	3	2	[3, 0]/1	[4, -2]	3	9
+v ² +v ⁴ +v ⁶	3	1	[3, 0]/1	[0, 4]	5	9
+v ² +v ⁴ +v ⁶	3	0	[3, 0]/1	[4, 0]	5	9
+v ⁴ +2v ⁶	2	4	[3, 2]/1	[3, 3]	7	10

$\mathrm{Sp}(4, \mathbb{R})$: $\nu = 0$

```
atlas> p
Value: final parameter(x=10,lambda=[2,1]/1,nu=[0,0]/1)
atlas> show_long (hodge_branch_std(p,10))
```

c	codim	x	lambda	hwt	dim	height
+1	0	10	[2, 1]/1	[0, 0]	1	0
+v ²	2	4	[1, 0]/1	[1, 1]	3	2
+v ²	3	3	[1, 0]/1	[-2, 2]	1	3
+v ²	3	2	[1, 0]/1	[2, -2]	1	3
+v+v ³	3	1	[1, 0]/1	[0, 2]	3	3
+v+v ³	3	0	[1, 0]/1	[2, 0]	3	3
+v ⁴	2	6	[2, 1]/1	[-1, 3]	3	6
+v ⁴	2	5	[2, 1]/1	[3, -1]	3	6
+v ² +2v ⁴	2	4	[2, 1]/1	[2, 2]	5	6
+v ³ +v ⁵	3	1	[2, 1]/1	[1, 3]	5	7
+v ³ +v ⁵	3	0	[2, 1]/1	[3, 1]	5	7
+v ³ +v ⁵	3	3	[3, 0]/1	[-2, 4]	3	9
+v ³ +v ⁵	3	2	[3, 0]/1	[4, -2]	3	9
+v ² +v ⁴ +v ⁶	3	1	[3, 0]/1	[0, 4]	5	9
+v ² +v ⁴ +v ⁶	3	0	[3, 0]/1	[4, 0]	5	9
+v ⁴ +2v ⁶	2	4	[3, 2]/1	[3, 3]	7	10

$\mathrm{Sp}(4, \mathbb{R})$: $\nu = \rho/3$

```
atlas> reducibility_points (p)
```

```
Value: [1/3,1/2,1/1]
```

```
atlas> show_long (hodge_branch_std(p*(1/3),10),x)
```

c	codim	x	lambda	hwt	dim	height
+1	0	10	[2, 1]/1	[0, 0]	1	0
+v	2	4	[1, 0]/1	[1, -1]	3	2
+v ²	3	3	[1, 0]/1	[-2, -2]	1	3
+v ²	3	2	[1, 0]/1	[2, 2]	1	3
+v+v ²	3	1	[1, 0]/1	[0, -2]	3	3
+v+v ²	3	0	[1, 0]/1	[2, 0]	3	3
+v ³	2	6	[2, 1]/1	[-1, -3]	3	6
+v ³	2	5	[2, 1]/1	[3, 1]	3	6
+v ² +2v ³	2	4	[2, 1]/1	[2, -2]	5	6
+v ² +v ⁴	3	1	[2, 1]/1	[1, -3]	5	7
+v ² +v ⁴	3	0	[2, 1]/1	[3, -1]	5	7
+v ³ +v ⁴	3	3	[3, 0]/1	[-2, -4]	3	9
+v ³ +v ⁴	3	2	[3, 0]/1	[4, 2]	3	9
+v ² +v ³ +v ⁵	3	1	[3, 0]/1	[0, -4]	5	9
+v ² +v ³ +v ⁵	3	0	[3, 0]/1	[4, 0]	5	9
+v ³ +2v ⁵	2	4	[3, 2]/1	[3, -3]	7	10

$\mathrm{Sp}(4, \mathbb{R})$: $\nu = \rho/2$

```
atlas> q
Value: final parameter(x=10,lambda=[2,1]/1,nu=[2,1]/2)
atlas> show_long (hodge_branch_std(q,10),x)
```

c	codim	x	lambda	hwt	dim	height
+1	0	10	[2, 1]/1	[0, 0]	1	0
+v	2	4	[1, 0]/1	[1, -1]	3	2
+v	3	3	[1, 0]/1	[-2, -2]	1	3
+v	3	2	[1, 0]/1	[2, 2]	1	3
+2v	3	1	[1, 0]/1	[0, -2]	3	3
+2v	3	0	[1, 0]/1	[2, 0]	3	3
+v^2	2	6	[2, 1]/1	[-1, -3]	3	6
+v^2	2	5	[2, 1]/1	[3, 1]	3	6
+3v^2	2	4	[2, 1]/1	[2, -2]	5	6
+v^2+v^3	3	1	[2, 1]/1	[1, -3]	5	7
+v^2+v^3	3	0	[2, 1]/1	[3, -1]	5	7
+v^2+v^3	3	3	[3, 0]/1	[-2, -4]	3	9
+v^2+v^3	3	2	[3, 0]/1	[4, 2]	3	9
+2v^2+v^4	3	1	[3, 0]/1	[0, -4]	5	9
+2v^2+v^4	3	0	[3, 0]/1	[4, 0]	5	9
+v^3+2v^4	2	4	[3, 2]/1	[3, -3]	7	10

$\mathrm{Sp}(4, \mathbb{R})$: $\nu = \rho/2$

```
atlas> q
Value: final parameter(x=10,lambda=[2,1]/1,nu=[2,1]/2)
atlas> show_long (hodge_branch_std(q,10),x)
```

c	codim	x	lambda	hwt	dim	height
+1	0	10	[2, 1]/1	[0, 0]	1	0
+v	2	4	[1, 0]/1	[1, -1]	3	2
+v	3	3	[1, 0]/1	[-2, -2]	1	3
+v	3	2	[1, 0]/1	[2, 2]	1	3
+2v	3	1	[1, 0]/1	[0, -2]	3	3
+2v	3	0	[1, 0]/1	[2, 0]	3	3
+v^2	2	6	[2, 1]/1	[-1, -3]	3	6
+v^2	2	5	[2, 1]/1	[3, 1]	3	6
+3v^2	2	4	[2, 1]/1	[2, -2]	5	6
+v^2+v^3	3	1	[2, 1]/1	[1, -3]	5	7
+v^2+v^3	3	0	[2, 1]/1	[3, -1]	5	7
+v^2+v^3	3	3	[3, 0]/1	[-2, -4]	3	9
+v^2+v^3	3	2	[3, 0]/1	[4, 2]	3	9
+2v^2+v^4	3	1	[3, 0]/1	[0, -4]	5	9
+2v^2+v^4	3	0	[3, 0]/1	[4, 0]	5	9
+v^3+2v^4	2	4	[3, 2]/1	[3, -3]	7	10

```
atlas> show_long (hodge_branch_std(trivial(G),10),x)
```

c	codim	x	lambda	hwt	dim	height
+1	0	10	[2, 1]/1	[0, 0]	1	0
+1	2	4	[1, 0]/1	[1, -1]	3	2
+1	3	3	[1, 0]/1	[-2, -2]	1	3
+1	3	2	[1, 0]/1	[2, 2]	1	3
+2	3	1	[1, 0]/1	[0, -2]	3	3
+2	3	0	[1, 0]/1	[2, 0]	3	3
+1	2	6	[2, 1]/1	[-1, -3]	3	6
+1	2	5	[2, 1]/1	[3, 1]	3	6
+2+v	2	4	[2, 1]/1	[2, -2]	5	6
+1+v	3	1	[2, 1]/1	[1, -3]	5	7
+1+v	3	0	[2, 1]/1	[3, -1]	5	7
+2v	3	3	[3, 0]/1	[-2, -4]	3	9
+2v	3	2	[3, 0]/1	[4, 2]	3	9
+3v	3	1	[3, 0]/1	[0, -4]	5	9
+3v	3	0	[3, 0]/1	[4, 0]	5	9
+2v+v^2	2	4	[3, 2]/1	[3, -3]	7	10

$\mathrm{Sp}(4, \mathbb{R})$: $\nu = \rho$

```
atlas> show_long (hodge_branch_std(trivial(G),10),x)
```

c	codim	x	lambda	hwt	dim	height
+1	0	10	[2, 1]/1	[0, 0]	1	0
+1	2	4	[1, 0]/1	[1, -1]	3	2
+1	3	3	[1, 0]/1	[-2, -2]	1	3
+1	3	2	[1, 0]/1	[2, 2]	1	3
+2	3	1	[1, 0]/1	[0, -2]	3	3
+2	3	0	[1, 0]/1	[2, 0]	3	3
+1	2	6	[2, 1]/1	[-1, -3]	3	6
+1	2	5	[2, 1]/1	[3, 1]	3	6
+2+v	2	4	[2, 1]/1	[2, -2]	5	6
+1+v	3	1	[2, 1]/1	[1, -3]	5	7
+1+v	3	0	[2, 1]/1	[3, -1]	5	7
+2v	3	3	[3, 0]/1	[-2, -4]	3	9
+2v	3	2	[3, 0]/1	[4, 2]	3	9
+3v	3	1	[3, 0]/1	[0, -4]	5	9
+3v	3	0	[3, 0]/1	[4, 0]	5	9
+2v+v^2	2	4	[3, 2]/1	[3, -3]	7	10

The principi series at ρ .

(Twisted) Endoscopy

G simple, ${}^\vee G$ its dual group, ${}^\vee \delta$ a non-trivial distinguished automorphism of ${}^\vee G$.

For $s \in {}^\vee G$ set ${}^\vee \tau = \text{int}(s) {}^\vee \delta$, assumed semisimple of finite order. Then

$${}^\vee H = (({}^\vee G)^{{}^\vee \tau})^\circ = \text{Cent}(s {}^\vee \delta)^\circ$$

H , the dual of ${}^\vee H$, is a **twisted endoscopic group** for G . (${}^\vee \delta = 1$: ordinary endoscopy.)

Algorithm: $(s, {}^\vee \delta)$: $({}^\vee G, {}^\vee \delta)$ has a **folded** affine Dynkin diagram whose finite part is the diagram of $({}^\vee G)^{{}^\vee \delta}$.

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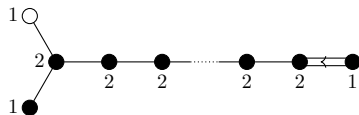
Algorithm: $(s, {}^\vee \delta)$: $({}^\vee G, {}^\vee \delta)$ has a **folded** affine Dynkin diagram whose finite part is the diagram of $({}^\vee G) {}^\vee \delta$. Then delete nodes from this diagram.

(Twisted) Endoscopy

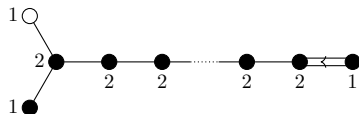
Atlas:

```
set endoscopic_group(  
  RootDatum G,      {G not G_vee}  
  mat delta_vee,    {optional dual twist, trivial for ordinary endoscopy}  
  mat xi_vee,       {dual involution defining the L-group of G; if inner class is  
                     given don't need to specify this}  
  ratvec s,         {take centralizer( $\exp(2i\pi s)$ )}  
  mat twist) =      {an additional twist; the identity gives the default  
                     inner class of the endoscopic group  
                     this is only needed to get other inner classes of H}
```

${}^\vee \mathbb{D}$: folded Dynkin diagram of type $A_{2m-1}^{(2)}$:

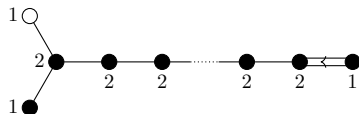


${}^\vee \mathbb{D}$: folded Dynkin diagram of type $A_{2m-1}^{(2)}$:



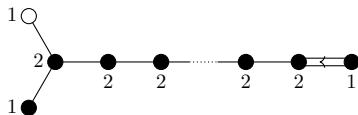
Basic case (order 2): Delete the affine node:

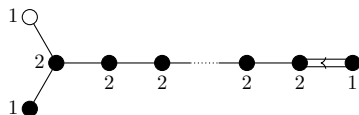
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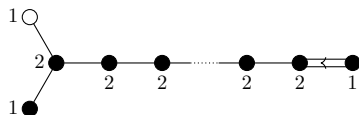
Basic case (order 2): Delete the affine node:

$$\begin{array}{ccc}
 G = \text{GL}(2m) & \longrightarrow & {}^{\vee}G = \text{GL}(2m) \\
 & & \downarrow \\
 H = \text{SO}(2m + 1) & \longleftarrow & {}^{\vee}H = \text{Sp}(2m)
 \end{array}$$



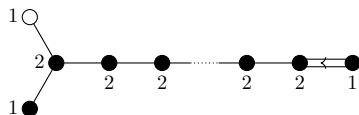


Other case(order 2): Deleting the long node (labeled 1)



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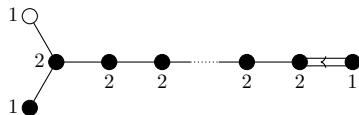
$$\begin{array}{ccc}
 G = \text{GL}(2m) & \longrightarrow & {}^\vee G = \text{GL}(2m) \\
 & & \downarrow \\
 H_0 = \text{SO}(2m+1) & \longleftarrow & {}^\vee H_0 = \text{Sp}(2m) \\
 \uparrow & & \\
 H = \text{SO}(2m) & \longleftarrow & {}^\vee H = \text{SO}(2m)
 \end{array}$$

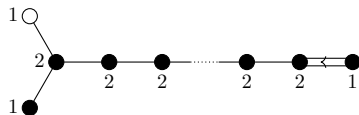


Other case(order 2): Deleting the long node (labeled 1)

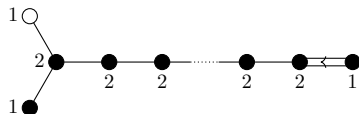
$$\begin{array}{ccc}
 G = \text{GL}(2m) & \longrightarrow & {}^\vee G = \text{GL}(2m) \\
 & & \downarrow \\
 H_0 = \text{SO}(2m+1) & \longleftarrow & {}^\vee H_0 = \text{Sp}(2m) \\
 \uparrow & & \\
 H = \text{SO}(2m) & \longleftarrow & {}^\vee H = \text{SO}(2m)
 \end{array}$$

In this example $H = \text{SO}(2m) \subset H_0 = \text{SO}(2m+1)$



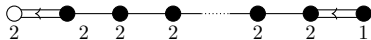


Other case (order 4): black node, label 2



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$$\begin{array}{ccc}
 G = \text{GL}(2m) & \longrightarrow & {}^v G = \text{GL}(2m) \\
 & & \downarrow \\
 H_0 = \text{SO}(2m+1) & \longleftarrow & {}^v H_0 = \text{Sp}(2m) \\
 \uparrow & & \\
 H = \text{SO}(2p) \times \text{SO}(2q+1) & \longleftarrow & {}^v H = \text{SO}(2p) \times \text{Sp}(2q)
 \end{array}$$



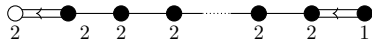
Basic case (order 2): short node label 2

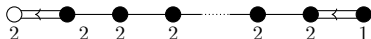
$$G = \text{GL}(2m + 1) \longrightarrow {}^\vee G = \text{GL}(2m + 1)$$



$$H = \text{Sp}(2m) \longleftarrow {}^\vee H = \text{SO}(2m + 1)$$

$$A_{\text{even}} \quad \text{GL}(2m+1)$$

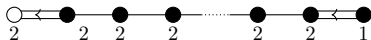




Other case (order 4): black node with label 2

$$\begin{array}{ccc}
 G = \text{GL}(2m) & \longrightarrow & {}^\vee G = \text{GL}(2m) \\
 & & \downarrow \\
 H_0 = \text{Sp}(2m) & \longleftarrow & {}^\vee H_0 = \text{SO}(2m+1)
 \end{array}$$

$$H = \text{SO}(2m+1) \longleftarrow {}^\vee H = \text{Sp}(2m)$$



Other case (order 4): black node labeled 2 gives

$$\begin{array}{ccc}
 G = \text{GL}(2m) & \longrightarrow & {}^\vee G = \text{GL}(2m) \\
 & & \downarrow \\
 H_0 = \text{Sp}(2m) & \longleftarrow & {}^\vee H_0 = \text{SO}(2m + 1)
 \end{array}$$

$$H = \text{SO}(2p + 1) \times \text{Sp}(2q) \longleftarrow {}^\vee H = \text{Sp}(2p) \times \text{SO}(2q + 1)$$

Note $H \not\subset H_0$

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endoscopy endoscopic stopped blocked fiber

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A pattern-matcher screening for medical or harm content, reading this without the mathematical context.

Visualizing the FPP

[digression]

Thank You

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